

Evacuation Simulations using Cellular Automata

Pablo Cristian Tissera, Marcela Printista*, Marcelo Luis Errecalde

Lab. de Investigación y Desarrollo en Inteligencia Computacional (LIDIC)[†]

Universidad Nacional de San Luis - Ej. de los Andes 950 - (D5700HHW) San Luis, Argentina

{*ptissera, mprinti, merreca*}@unsl.edu.ar

Abstract

Computer simulations using Cellular Automata (CA) have been applied with considerable success in different scientific areas, such as chemistry, biochemistry, economy, physics, etc. In this work we use CA in order to specify and implement a simulation model that allows to investigate behavioral dynamics for pedestrians in an emergency evacuation. In particular, we will concentrate on those cases that involve the forced evacuation of a large number of people due to the threat of the fire, within a building with a specific number of exits. The work includes a brief introduction to the main concepts of CA that were considered for implementing the simulation model. As support of the model, a new simulation system named EVAC is presented which allows to design, construct, execute, visualize and analyze different configurations of the building to be evacuated. The experimental work allows to identify important safety aspects to be considered at the time of designing a building, to detect the strengths of the CA approach when used as simulation tool and to suggest possible extensions that would allow to represent some particularities of the problem in a more suitable way.

Keywords: Computer Simulation, Cellular Automata, Behavioral dynamics for pedestrians, Building Evacuation.

1 Introduction

Shopping centers, schools and dance halls are some examples of buildings commonly used in different daily activities that involve the meeting of a large number of people within a closed area. The designers of these types of building usually attempt to maximize the productivity of the available space, but also it is necessary to consider a suitable planning for assuring people safety when an unusual behavior of the crowd occurs. One of the most frequent causes of this kind of behavior is the emergency evacuation due to the threat of fire. In such situation, a closed area, with a relatively small number of fixed exits, must

be evacuated for a large number of people.

In the last years, the interest in models of emergency evacuation processes and pedestrian dynamics has increased. Models used for evaluating the evacuation processes can broadly be categorized in microscopic and macroscopic approaches. The macroscopic approaches are based on differential equations that take into account the similarities with systems previously studied like dynamics of fluids. They are considered as black boxes because they do not allow to analyze its internal procedures. On the other hand, the microscopic approaches allow to consider how the system state evolves during the model running. In this context, the importance of simulation models for pedestrian dynamics in emergency evacuations is out of discussion. Experimental research in the real world has ethical, financial and logical limitations; furthermore, an evacuation exercise also exhibits some of the risks that we could find in a real evacuation. The simulation allows to specify different scenes with a large number of people and environmental features, making easier the study of the complex behaviors that arise when the people interact.

The cellular automata (CA) are discrete dynamic systems that offer an appealing alternative to deal with this kind of situations due to their capacity to develop complex behaviors from a simple set of rules. Basically, these rules allow to specify the new state of a cell based on the state of the neighboring cells. In this way, it is possible to model complex dynamic systems from the specification of the local dynamics of its components. A further advantage of these systems is the support usually provided for displaying the results in a graphical way, allowing an easier comprehension of the dynamics of the system under study.

When used as simulation tools, the CA have demonstrated to be very useful at the time of constructing artificial scenes, mainly in those domains where other approaches are not suitable.

Taking into account the previous aspects we decided to use the CA as basis of our simulation model and to investigate the pedestrian dynamics

in emergency situations. In particular, we concentrate on those cases that involve the forced evacuation of a large number of people due to the threat of the fire, within a building with a specific number of exits.

This work has the following *objectives*:

- To analyze the strengths and limitations of AC for modeling this type of domains.
- To present a simulation tool that allows to design, construct, execute, visualize and analyze different configurations of the building to be evacuated.
- To evaluate the performance of forced evacuation under different fire conditions. This evaluation is important to achieve a better understanding of the evacuation process and this way, to identify relevant aspects to be considered in the design of new buildings.

In section 2 we include a brief introduction to the main concepts of the AC that were used in the simulation model presented in section 3. In section 4 we describe our experimental work with different instances of the problem at hand and report the performance analysis of each case. This section also includes a few commentaries on the operation of the EVAC simulation system that supports the proposed model. Finally, the section 5 discusses the strengths and limitations of our approach and possible future extensions.

2 Cellular Automata

The *Cellular Automata (CA)* arose as result of the joint work of John Von Neumann and Stanislaw Ulam in studies related to machines with auto-replication capabilities. These systems received special attention when Jhon H. Conway proposed in 1970 an automaton known as the *game of life* and later popularized by Martin Gardner in an article of *Scientific American*. Since then the CA have grown in popularity within of the scientific community and some researchers have even claimed that the CA represent a possible new paradigm of the physics field named *Digital Mechanics*.

The CA are mathematical systems with discrete values in space, time and state. In addition to have *auto-replication* and *universal computation* capabilities, the CA can generate extreme ordered behaviors from a total disorder (*auto-organization* effects). This last property plays a very important role at the time of explaining certain kind of behaviors observed in physical and biological phenomena [1, 2]. The CA have been used to create digital models of physics universes

that simulate different phenomena as chemical reactions, diffusion processes, hydrodynamic, mechanic, filtration, chaos theory and others. On the other hand, some elemental organisms obey simple local rules that can be described in a direct way by means of CA.

With respect to the structure of a cellular automaton it can be considered as a dynamic system that represents a grid of locally connected finite automata [3]. Each automaton produces an output from several inputs, modifying its state in this process by means of a transition function. The state of a cell of a cellular automaton in a particular generation only depends of the states of its neighboring cells and the state the cell had in the previous generation.

At the moment, it does not exist an universal consensus about the terminology and definitions related to the CA. For this reason, we will introduce some elemental concepts that will help the reader to a better understanding of this work. Intuitively, we will consider a cellular automaton as a system composed by a array of cells A . Each cell c_i in A represents a *finite automaton* with a set of states Q , an input alphabet Σ and a transition function $\delta : Q \times \Sigma \rightarrow Q$. A particularity of this automaton is that the input alphabet Σ is given by all the possible combination of the *cell states* of the adjacent (neighboring) cells. If we denote N^{-c_i} to the set of cells that we will consider as neighbor of an arbitrary cell c_i , and $|N^{-c_i}| = n$ is the number of adjacent cells, then the input alphabet will be $\Sigma \equiv Q^n$. Usually, a cell c_i and its adjacent cells are all together considered and represented as a unique set $N = \{c_i\} \cup N^{-c_i}$ which is referenced as the *neighborhood*. It is important to emphasize that each cell has a copy of the *structure* of the finite automaton, but the initial state of each cell could be different.

We can now to define a *Cellular Automaton* as a 4-tuple $M = \langle A, Q, \delta, N \rangle$ where:

- A is a D -dimensional array, and each component (cell of the array) has associated a *finite automaton*.
- Q is a finite set of states (of the automaton) of a cell
- N is the specification of which cells are included in a neighborhood, $N \equiv \{c_i\} \cup N^{-c_i}$ such that N^{-c_i} are the adjacent cells to c_i .
- Let $\Sigma \equiv Q^n$ where $n = |N^{-c_i}|$ is the number of adjacent cells to c_i . The *transition function* of states, $\delta : Q \times \Sigma \rightarrow Q$, is a mapping such that if $q_i \in Q$ is the state of the cell c_i in the time t and $q_{i+1}, q_{i+2}, \dots, q_{i+n} \in \Sigma$ are

the states of the adjacent cells¹ to c_i , and

$$\delta(q_i, q_{i+1}, q_{i+2}, \dots, q_{i+n}) = q'_i$$

then q'_i denotes the state of c_i in the time $t + 1$.

It is usual to refer to the cell c_i under consideration as the “central cell” or “cell 0” and to enumerate as “neighbor 1”, . . . , “neighbor n ” the adjacent cells to c_i . The δ function is usually represented in tabular form with $|Q|^{n+1}$ rules:

$$\delta(q_i, q_{i+1}, q_{i+2}, \dots, q_{i+n}) \rightarrow q'_i$$

where $q_i, q_{i+1}, q_{i+2}, \dots, q_{i+n}$ are the states of the central cell and its neighbors at the time t and q'_i is the state of the central cell at the time $t + 1$. In some cases, it is possible to specify *probabilistic transition rules*, where an arbitrary probability p can be associated to a transition rule. The semantic of this kind of rules establishes that, always that a cell matches the configuration of the specified neighborhood in a probabilistic rule, the cell will have at time $t + 1$ the new state specified in such rule with probability p .

Example: the term *Elementary Cellular Automaton (ECA)* refers to a class of CA that were carefully studied by Stephen Wolfram. An ECA is composed by a one-dimensional array A , where each cell of the array can be in one of two possible states ($Q = \{0, 1\}$), and the neighborhood is defined as $N = \{c_{i-1}, c_i, c_{i+1}\}$. Since $|N| = 3$, δ will have $2^3 = 8$ possible transition rules. Below, we show the specification² of some rules of an ECA proposed by Stephen Wolfram:

1. If a cell at the time t is inactive (0), is activated at the time $t + 1$ if some of the adjacent cells (left or right) is active (1).
2. An active cell at the time t , is turned inactive at the time $t + 1$ if its adjacent cells are both actives or both inactives.
3. In other case a cell preserve its previous state.

111	110	101	100	011	010	001	000
0	1	0	1	1	0	1	0

Figure 1: Graphical representation of the rules.

¹According to N^{-c_i} .

²We specify the rules in an informal way in order to achieve a more comprehensive reading. Nevertheless, it has to be observed that each one of these rules specified in natural language can represent several rules $\delta(q_i, q_{i+1}, q_{i+2}) \rightarrow q'_i$.

In Figure 1 a graphical representation of these transition rules is presented. In Figure 2 the evolution of this ECA is shown. It is interesting to observe that the fractal generated in this case corresponds to the “Sierpinski’s triangle”.

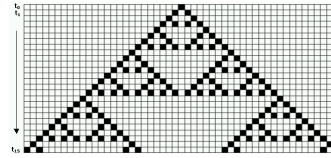


Figure 2: Evolution of an ECA.

3 Model Description

Problem solving related to activities in the real world is usually handled with simulation models, which are created to reflect the main features of the real system under consideration. Below, we describe the main features of our proposed model which is based on a CA approach.

Cellular Space: it is a finite bi-dimensional array (grid) with closed boundaries. Each cell of the cellular space represents a 40 × 40 centimeters square.³ The dimensions of the cellular space are specified in meters. So, one grid of 10 × 10 meters will contain 25 cells by side (Figure 3).

States: a cell can be in one of the states of the set $Q = \{W, E, P, O, S, SF, PS, PSF\}$ where:

- | | |
|---|--|
| <ul style="list-style-type: none"> W : External wall cell. E : Empty cell. P : Cell with a person. O : Internal obstacle cell. S : Cell with smoke. | <ul style="list-style-type: none"> SF : Cell with smoke and fire. PS : Cell with a person and smoke. PSF : Cell with a person, smoke and fire. |
|---|--|

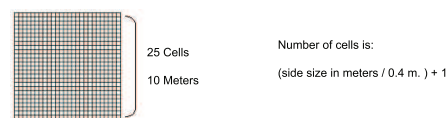


Figure 3: Cellular space of 10 × 10 meters.

Neighborhood: the neighborhood considered in the model is *Moore’s Neighborhood* (Figure 4), that includes the eight cells surrounding the central cell. With this choice we aim to provide to each individual in the system with all possible movement directions.

³This is the space usually occupied by a person in a crowd with maximal density [4, 5, 6].

Initial Configuration: before the simulation starts it must be specified diverse information related to the outer walls, inner obstacles, individuals, combustible locations, cell with fire and arrangement of the exits. In EVAC this task can be realized by means of its graphical interface.

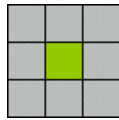


Figure 4: Moore's Neighborhood

Virtual clock: taking into account recent studies [4, 6, 7], an updating time of 0.3 seconds by time step was specified for our model. This value is the estimated time required by a pedestrian for walking 0.4 meter (size of a cell side).

Model Evolution Rules:

1. Rules about the *building*: a cell in state *W* or *O* (outer wall or obstacle) will not change its state throughout the simulation.
2. Rules about *smoke propagation*: a cell with smoke⁴ in time *t*, also will have smoke in time *t*+1. If at time *t* the central cell does not have smoke, but some of its adjacent cells have smoke, the central cell also will have smoke at time *t* + 1 with a probability proportional to the number of adjacent cells with smoke. For example, in Figure 5 the central cell (marked with a *X*) has four adjacent cells with smoke on a total of eight adjacent cells. In this case, the central cell will have smoke in the next time step with probability $\frac{1}{2}$.
3. Rules about *fire propagation*: these rules are analogous to the rules for smoke propagation. However, they incorporate an additional constraint: a non-zero combustion level of the cell is required.
4. Rules about the *people motion*: a cell without people will have a person in the following cycle if *a*) at least one adjacent cell contains an individual and *b*) the distance from the cell under consideration to an exit is less than the distance from the cell occupied by the individual to the exit. In other cases, the cell does not change its state. Other aspects related to the people motion are described in subsection 3.1.

⁴In one of the following states: *S*, *SF*, *PS* or *PSF*.

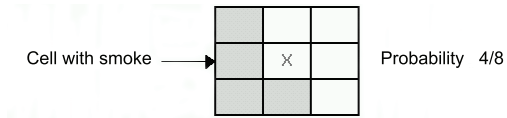


Figure 5: Probability of smoke propagation.

3.1 People motion

Two important aspects must be considered when simulating the movement of people: *a*) *estimation of distances* from the cells to an exit and *b*) *handling of collisions* between individuals. For the first problem, the Dijkstra's algorithm was used. This algorithm solves the single-source shortest-paths problem on a weighted graph [8]. The cellular space is considered as a graph, where each cell represents a node and all the edges connecting adjacent cells have weight 1. Cells with state *W* (wall) or *O* (obstacle) are not considered to build the graph.⁵ If the building have more than one exit, the distance computation take into account the exit nearest to the cell (see Fig. 6).

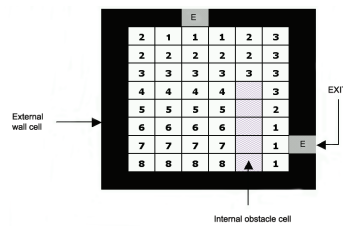


Figure 6: Estimation of distances to the exits.

The handling of collisions between the individuals, is one of the most common problems faced in pedestrian dynamics. The conflict arises when two or more people simultaneously attempt to occupy the same physical location belonging to an exit way. To solve this aspect, we changed the approach commonly used in other works. Instead of being the pedestrian who decides which cell to move, the current cell⁶ is in charge of choosing between the people in the adjacent cells, which pedestrian will move to the cell.

The obvious question is: Which are the criteria used by the cell in order to select the pedestrian to move in? The process that we used for solving this point is the following: *a*) any person in the neighborhood whose distance to an exit is shorter than the distance of the cell attempting to receive an individual, will not be considered as candidate

⁵An individual can not cross a wall and therefore he should surround the wall to reach the exit

⁶If the cell is in a state suitable to receive an individual.

(see Figure 7). *b*) If more than one individual still remains as candidate to occupy the cell, the one with the minor number of damage points (parameter specified in the model) will be selected. *c*) Finally, if the conflict persists, the person will be selected at random.

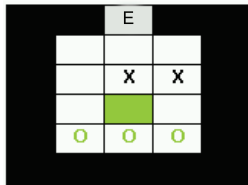


Figure 7: Shadow cell will choose one person from some cell *O*, because the cells with an *X*, are located in a better location (nearer to exit.)

4 Experiments

The experiments were carried out with EVAC, an integrated simulation system based on cellular automata. EVAC is a system developed in Java that allows the design and simulation of spatial environments in an explicit way. EVAC simulator offers a friendly graphical interface which can be easily used by non expert users.

EVAC can be used in two different modes: *edition* mode and *execution* mode. In edition mode, it is possible to create a new simulation model or to modify an already existent model. The general parameters can be set and the main features of the building to be used during the simulation can be graphically specified. In execution mode in turn, an animation of the evacuation process is displayed to the user using the model previously defined in the edition mode. This mode also generates a report with statistical information gathered during the simulation of the evacuation process. These facilities are introduced in EVAC as useful tools that allow the user a better comprehension, design and analysis of the study case under consideration.

In each simulation step, EVAC performs an exhaustive updating of the cellular space, using at this end, the evolution rules described in section 3. The cells of the CA are updated at random with the purpose to avoid any sort of bias generated by the order in which the cells are updated.

The simulation finishes when all the people have evacuated the building. With the purpose of obtaining acceptable statistical data, the results shown below corresponds to the averaged results of 100 independent replications of each experiment. The corresponding confidence intervals

were obtained, for the mean evacuation time and mean traveled distance per person to the exit.

The experiments conceptually were divided into three groups, that correspond to each one of the rows with environment configurations shown in Figure 8. The first group of environments (*A*, *B*, *C* and *D*) addressed those situations where one single exit in the building to be evacuated exists, varying the different occupation densities of the environment (number of people) and the size of the exit. With the group of environments *E*, *F*, *G* and *H* the influence of obstacles in the evacuation process was analyzed. First, the presence of an obstacle of great size was considered and then this obstacle was divided into two smaller separated obstacles. The last group of environments represents different variants and combinations of the two previous groups of experiments. In the case of the environments *I* and *J*, they represent the variants with two exits of the cases *C* and *D*. The environments *K* and *L* locate the exit and the people in opposite corners. In the first case, the environment does not have divisions, leaving clear the path to the exit; in the environment *L*, on the other hand, a series of divisions force the individuals to follow a sequence of walkways until arriving to the exit.

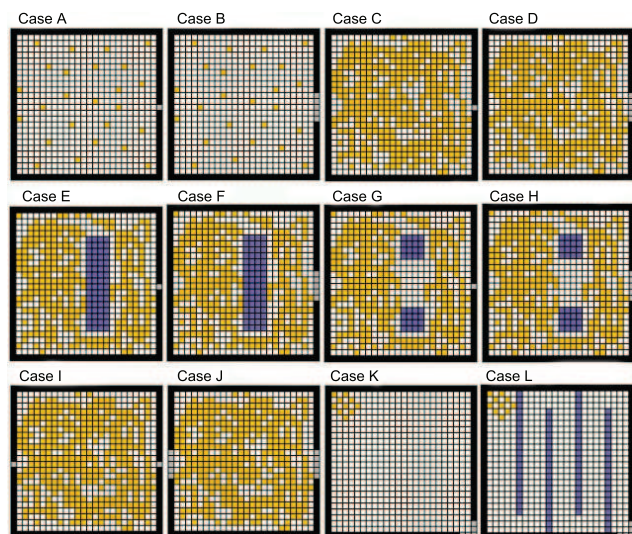


Figure 8: Groups of Experiments.

4.1 Results and analysis

Table 1 summarizes the results obtained with the three groups of experiments analyzed below.

4.1.1 First group of experiments

The results in this case confirm our intuition that the evacuation times of the environment *A* would

Case	Ind.	Size of exit	Mean evac. time	Mean evac. time per person	Mean trav. distance per person
A	25	0.4	16.10	8.06-8.11	10.73-10.82
B	25	2	15.02	7.24-7.28	9.64-9.72
C	335	0.4	104.23	11.02-11.03	14.70-14.71
D	335	2	35.18	7.718-7.72	10.28-10.29
E	268	0.4	83,64	13.02-13.03	17.36-17.37
F	268	2	33,27	9.52-9.53	12.70-12.71
G	268	0.4	83,60	11.16-11.17	14.88-14.90
H	268	2	29,03	8.249-8.25	10.99-11.01
I	335	0.4	54,73	7.35-7.36	9.81-9.82
J	335	2	20,73	4.78-4.79	6.37-6.38
K	10	2	21,65	15.90-16.15	21.15-21.60
L	10	2	61,73	52.69-53.41	70.09-71.38

Table 1: Results of the experiments.

be greater than the required times for evacuating the environment B which has an exit of greater size. Nevertheless, an interesting result was the decrease in the mean traveled distance in the environment B regarding to the environment A . With the help of the animation module, was possible to determine that in A , the individuals move around the exit until they leave the building and this behavior results in a greater traveled distance. In the environment B , on the other hand, the majority of the individuals can access the exit in a direct way due to its greater size. This behavior will likely occur in a real evacuation, since it is not usual that the individuals when do not have direct access to an exit remain quiet in the same place. An aspect to note is that the increment of the exit size in the environment B contributes to a better performance but these improvements were not significant. In order to analyze the hypothesis that this behavior was caused by the low occupation density of the environment, the experiments C and D were carried out. The results obtained with these experiments show that the evacuation time obtained in the environment D is less than the half of the required in the environment C . As we supposed, the main cause of this behavior is the considerable increment in the number of individuals included in the model. Considering these results we can infer that the size of the exits has an important impact in the evacuation times only in those cases in which the quantity of individuals involved in the evacuation is significant, which resembles the behavior observed in a real situation.

4.1.2 Second group of experiments

If we observe the results for the cases E and F we can note an increment in the mean evacuation times per person and mean traveled distance, with respect to similar cases previously considered where the presence of obstacles has not been considered (environments C and D). This increment is due to the need of the pedestrians for surrounding the obstacle to arrive to the exit. We

have to note that although it is possible to observe a decrease of the total times of evacuation of the environments E and F respect to the environments C and D , this result is originated by a minor quantity of individuals in the environments E and F . The data obtained in the cases G and H show that the differences are notorious in the mean traveled distance as well as in the mean evacuation time per person. These results can be justified by the fact that the pedestrians do not need to surround the central obstacle (like in E and F cases). An interesting result obtained with this group of experiments arose from comparing statistical data gathered with environments E and G . Little variation was observed in the mean evacuation times due to a particular behavior produced near to the exit that we will denote as “bottle neck effect”. This occurs in those situations where the crowd moves toward the exit and the pedestrians gradually occupy the totally of the exit zone. Due to the small size of the exit, the only difference between both situations is determined by the time in which the pedestrians arrive and occupy totally the exit zone. In the slower case (E), this event approximately occurs in the middle of the evacuation process. So, we can conclude respect to this point that if we do not enlarge the exit size the only effect achieved is a faster crowding near to the exit. Finally, it is necessary to emphasize that a significant difference regarding to the prior situation was observed in the mean evacuation times of the environments F and H , due to a decrease in the impact of the “bottle neck effect” motivated by the presence of a larger exit.

4.1.3 Third group of experiments

The environments I and J are the result of incorporating a new exit in the opposite corner of the already existing exit in the environments C and D . By means of the visualization module provided by EVAC, it was possible to verify that in the cases I and J , the individuals form two groups of similar size and each group moves in direction to the closer exit. As expected, in this case was possible to observe an important decrease in all measures respect to the C and D cases. Obviously these improvements can be attributed to the modifications realized. Considering now the cases K and L , it is direct to verify that in the first case the evacuation times and mean traveled distances obtained, are notably lower than the values obtained with the second approach, because in the last one the pedestrians have to walk a larger distance.

When these groups of experiments were concluded, we decided to obtain a more detailed view of how important is the exit size and, at this

end, we used the environment shown in figure 9 (left). In this case, the same environment was used but different exit sizes were considered. As expected, the evacuation time decreases when we use a larger exit; however if we observe the results obtained when we increment the number of exits (cases *I* and *J*) we can infer that this last aspect has a more important impact in the speedup of the evacuation process.⁷

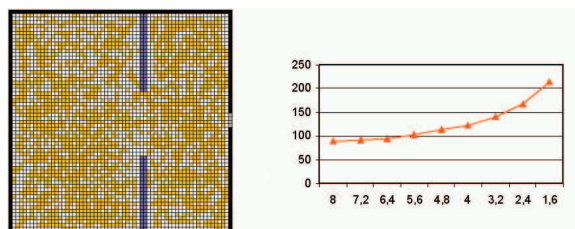


Figure 9: Exit size vs. evacuation times.

5 Conclusions

In this work, we used Cellular Automata for developing and implementing a simulation model of emergency evacuations due to the fire threat. In order to implement the model, we presented the EVAC system which allowed to analyze evacuation processes with different building configurations and experimental conditions. These studies were intended to detect which modifications in the design of buildings would improve the evacuation processes.

In spite of the assumptions introduced for obtaining a more simple model, the CA resulted to be very suitable tools for modeling this class of problems achieving in many cases, results very similar to those expected to occur in a real evacuation situation.

CA allowed to generate “local” and “uniform” behaviors that resemble the dynamics observed in real processes of fire and smoke propagation. However, these local features are not suitable for representing certain aspects of people behavior that require a more global and differentiated perspective. At the present time we are developing a hybrid model where the dynamics of fire and smoke propagation are modeled by means of CA and for simulating people behavior we are using goal oriented *intelligent agents* [10].

At present, our model does not take into account structural dynamic changes that could occur during the evacuation process, for instance,

⁷The interested reader is encouraged to consult [9] in order to obtain additional information about the EVAC system and other experiments carried out by the authors.

the creation of a new exit as consequence of shattering a window. In the future we will extend the model for supporting this capability and besides we will offer a parallel high performance simulation model.

Finally, we believe that the EVAC system is a good start point for analyzing and designing preventive safety policies and therefore, we suggest to the interested reader to contact the authors to obtain the Java source code of this system.

6 Acknowledgments

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